

Σ LIKE TERM IN PION ELECTROPRODUCTION NEAR THRESHOLD

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Abstract

Sum of the time component of Σ term and the induced pseudo scalar term in axial current is shown to be the t-channel pion pole in Born terms for pion electroproduction near threshold. We also show that this Σ term represents the charged pseudo scalar quark density matrix elements in nucleon and manifests itself in the L_0^+ amplitude on this reaction.

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I. INTRODUCTION

So far most of the theoretical frameworks to describe the pionproduction near threshold by electron or photon is usually based on Born amplitude [1,2]. It consists of the contributions of all poles to the invariant amplitude, but discarding the contributions associated with branch cuts, i.e. the resonances and/or the multi-particle intermediate states [1], thus it corresponds to the single particle intermediate states i.e. s-, u- channels mediated through nucleon, and t-channel through pion. These diagrams can be calculated by exploiting the effective lagrangians for the interactions with a constraint of gauge invariance. The classical Low Energy Theorem (LET) [2,3] for pion photoproduction can be obtained this way. Of course one needs to take the higher order contributions into account to obtain better theoretical results consistent with the recent experimental data [4,5] as in the chiral perturbation theory [6,7].

On the other side a current algebra (CA) approach [9,10] is alternative to this reaction. It enables us to derive the classical LET with the help of soft pion limit and the extrapolation to physical pion region. But the relationship between these two approaches still have a flaw : one could not derive explicitly the t-channel pion pole despite of many trials [2,8,11,12] although the s- and u-channels can be deduced from CA approach. Nevertheless it does not cause any problem in pion photoproduction because the t-channel pion pole (we call it t-pole) does not contribute at the pion threshold. Consequently the absence of t-pole in CA does not affect its predictive power at all. However in case of electroproduction it survives even at the pion threshold due to the coulomb polarization of virtual photon and contributes to longitudinal amplitudes.

Therefore one needs more careful analyses to apply CA method or Born approximation to pion electroproduction. Originally in CA approach one has two pole structures related to pion [2,11]. One of them is the induced pseudo-scalar (PS) part in the expectation value of axial current (call it as h-pole). Another pole appears from the time component of Σ term (σ -pole). This Σ term is called as chiral symmetry breaking term due to its operator

structure similar to the σ term in π -N scattering, and has two components, time and spatial parts.

The concept of Σ term in pionproduction is originated by Furlan *et al.* [14] and exploited by Nath and Singh [15] to show that its spatial component can be attributed to the discrepancy between the old experimental data [18] and the classical LET for π^0 production. But the evaluations of the spatial part of this Σ term, proportional to the commutator $[D^a(x), V_i(0)]$ with $D^a(x) = \int d^3x \partial^\mu A_\mu(x)$, turned out to be strongly model dependent as discussed in [16,17]. Our focus in this report, however, is not on its spatial part but on its time part. Here it should be reminded that both parts are fully independent because this Σ term is not self gauge invariant. Moreover the time part can be calculated model independently, and has a pion pole structure (σ -pole). Thus it affects pion electroproduction. Of course both σ - and h-poles in CA approach are entangled in some way to the pion pole in Born terms i.e. t-channel pion pole.

But the relations between the poles in these two different approaches are not clear yet [10]. Of course such a problem could be senseless if one considers the duality [19,20], supposed to be working in relatively high energy region, by which the summation of s-channels equates the t-channel summation. However if we remind that Born approximation has been successful to explain pionproduction with aid of effective lagrangians approach one needs to theoretically identify the t-pole.

In section II by rederiving a transition amplitude for pionproduction using real pion limit in order to keep the self consistency the existence of Σ term is confirmed. Secondly it is emphasized that around threshold the sum of σ -pole and h-pole in CA approach equals to the t-channel pion pole in Born terms. It means that the time component of the Σ term originated from the explicit chiral symmetry breaking effects manifests itself as a part of t-pole, thus appears as a part of longitudinal multipole amplitudes in pion electroproduction. More detailed calculation shows that each half of t-pole contribution to L_0^+ amplitude is attributed to σ - and h- pole contributions, respectively. Finally simple estimation of this effect using the experimental form factors is given. In section III, it is shown that the time

component of this Σ term could give us valuable informations about the charged PS quark density distribution in nucleon, as like σ term [21] in $\pi - N$ scattering, which shows scalar quark density distribution in nucleon. In section IV we show how large it affects the L_0^+ amplitude and discuss about the possible methods to extract Σ term from the experimental L_0^+ amplitude in pion electroproduction. A brief summary is done at the final section.

II. DERIVATION OF Σ LIKE TERM FROM CA MODEL

A. Transition amplitude for electropion production

We demonstrate our transition amplitude for electro- and photo-pionproduction using real pion limit and derive the reciprocal relations between the σ -pole and h-pole embedded in our transition amplitude and the t-pole in Born amplitude. We start from the hypothesis of partial conservation of axial current (PCAC),

$$\partial^\mu A_\mu^a = -f_\pi m_\pi^2 \phi^a \quad , \quad A_\mu^a = g_A \bar{\Psi} \gamma_\mu \gamma_5 \frac{\tau^a}{2} \Psi + f_\pi \partial_\mu \phi^a \quad , \quad (1)$$

where f_π is a pion decay constant and m_π^2 is the square of pion mass. The $f_\pi \partial_\mu \phi_\pi^a$ is the axial current from pion field satisfying the following Klein Gordon equation with nucleon source function,

$$(\square + m_\pi^2) \phi^a = -ig_{\pi NN} \bar{\Psi} \gamma_5 \tau^a \Psi. \quad (2)$$

The transition amplitude \mathcal{M}_ν^a for pionproduction $V_\nu^\gamma(k) + N(p_1) \rightarrow \pi^a(q) + N(p_2)$, where a is the cartesian isospin (we distinguish the pion charge as Greek letter) and ν is the polarization index, is written in the following way within the Bjorken-Drell convention :

$$\begin{aligned} \mathcal{M}_\nu^a &= \int d^4x e^{iqx} (-q^2 + m_\pi^2) \langle p_2 | T[\phi_\pi^a(x) V_\nu^\gamma(0)] | p_1 \rangle \\ &= \left(\frac{q^2 - m_\pi^2}{m_\pi^2 f_\pi} \right) \int d^4x e^{iqx} \langle p_2 | T(\partial_\mu A_\mu^a(x) V_\nu^\gamma(0)) | p_1 \rangle \\ &= \left(\frac{q^2 - m_\pi^2}{m_\pi^2 f_\pi} \right) \int d^4x e^{iqx} \langle p_2 | \partial_\mu T(A_\mu^a(x) V_\nu^\gamma(0)) - \delta(x_0) [A_0^a(x), V_\nu^\gamma(0)] | p_1 \rangle . \end{aligned} \quad (3)$$

Let us define C_ν^a and $T_{\mu\nu}^a$ as follows :

$$\begin{aligned} C_\nu^a &= \int d^4x e^{iqx} \delta(x_0) \langle p_2 | [A_0^a(x), V_\nu^\gamma(0)] | p_1 \rangle \\ T_{\mu\nu}^a &= i \int d^4x e^{iqx} \langle p_2 | T[A_\mu^a(x) V_\nu^\gamma(0)] | p_1 \rangle . \end{aligned} \quad (4)$$

Then,

$$\begin{aligned} f_\pi \mathcal{M}_\nu^a &= \frac{m_\pi^2 - q^2}{m_\pi^2} (C_\nu^a + q_\mu T_{\mu\nu}^a) \\ &= (C_\nu^a + q_\mu T_{\mu\nu}^a) - \frac{q^2}{m_\pi^2} (C_\nu^a + q_\mu T_{\mu\nu}^a) \\ &= \mathcal{M}_\nu^a(B) + \mathcal{M}_\nu^a(C) . \end{aligned} \quad (5)$$

Here the 2nd term $\mathcal{M}_\nu^a(C)$ does not contribute if one takes soft pion limit $q^2 \rightarrow 0$. The classical LET in pion photoproduction can be obtained from $\mathcal{M}_\nu^a(B)$ i.e. the nucleon pole terms can be obtained from $q^\mu T_{\mu\nu}^a$ using the N and $N\bar{N}N$ intermediate states [27,28] and the Kroll-Rudermann term, derived from the minimal coupling of πNN interaction in Born term, comes from C_ν^a . With Goldberg-Treimann (GT) relation $g_A M = f_\pi g_{\pi NN}$ and the equal time commutator (ETC) of axial charge and vector current one obtains the following result :

$$C_\nu^a = \bar{u}(p_2) \frac{I_a}{2} [G_A(t) \gamma_\nu \gamma_5 + \frac{G_P(t)}{2M} (k - q)_\nu \gamma_5] u(p_1) , \quad (6)$$

where the momentum transfer is given by $t = (k - q)^2$ and $I_a = i\epsilon_{a3b}\tau_b$. The 2nd term, peculiar to CA approach, is the induced pseudo-scalar term. This does not contribute to pion photoproduction, but makes an important contribution to pion electroproduction as will be shown later on.

If we adopt the real pion limit $\mathcal{M}_\nu^a(C)$ gives the two terms :

$$\begin{aligned} \mathcal{M}_\nu^a(C) &= -\frac{1}{m_\pi^2} \int d^4x \partial_0 e^{iqx} \langle p_2 | \delta(x_0) [\partial_\mu A_\mu^a(x), V_\nu^\gamma(0)] | p_1 \rangle \\ &\quad + \partial_\alpha \int d^4x e^{iqx} \langle p_2 | T(\partial^\alpha f_\pi \phi^a(x) V_\nu^\gamma(0)) | p_1 \rangle . \end{aligned} \quad (7)$$

The 1st term leads to the following contribution, which is called as the chiral symmetry breaking term because it would go to zero in chiral limit,

$$\frac{iq^0}{m_\pi^2} \Sigma_\nu^a(\gamma^* \pi^a) = \frac{iq^0}{m_\pi^2} \int d^4x e^{iqx} \delta(x_0) \langle p_2 | [\partial_\mu A_\mu^a(x), V_\nu^\gamma(0)] | p_1 \rangle . \quad (8)$$

Addition of the 2nd term in $\mathcal{M}_\nu^a(C)$ to $q_\mu T_{\mu\nu}^a$ in $\mathcal{M}_\nu^a(B)$ yields

$$iq^\mu \tilde{T}_{\mu\nu}^a = iq^\mu \int d^4x e^{iqx} \langle p_2 | T[\bar{A}_\mu^a(x) V_\nu^\gamma(0)] | p_1 \rangle , \quad (9)$$

where $\bar{A}_\mu^a = A_\mu^a(x) - f_\pi \partial_\mu \phi^a(x)$ is the axial currents with the pion axial current subtracted.

As a result one does not retain the pion pole structure in $iq^\mu \tilde{T}_{\mu\nu}^a$ any more.

Our transition amplitude is summarized as follows :

$$\mathcal{M}_\nu^a = \frac{1}{f_\pi} C_\nu^a + i \frac{q_\mu}{f_\pi} \tilde{T}_{\mu\nu}^a + \frac{iq^0}{m_\pi^2 f_\pi} \Sigma_\nu^a(\gamma^* \pi^a) . \quad (10)$$

This amplitude is identical to Weise's result [11], which is derived from Ward-Takahasi identity. But we used the real pion limit [12]. The classical LET in photopion production can be derived from the 1st and 2nd term similarly to the case in soft pion limit. The spatial part of the last term, $\Sigma_{\nu=space}^a$ term, are said to contribute to E_0^+ amplitude [11] in photopion production. For example, the calculation of Nath and Singh [15] showed some interesting effects by using a free quark algebra, but it was strongly criticized by Kamal [16], and by Bernstein and Holstein [17] in the following respects. The commutator $[D^a(x), V_i(0)]$ in Σ_{space}^a should be zero if the model used satisfies PCAC. Without considering the contributions from the anomalous dimensions at the loop level due to the gluonic contributions, the axial current by the free quark algebra does not satisfy PCAC. Even the free quark calculation shows some model dependent results. The calculation [11] of Σ_{space}^a by chiral nucleon model showed more or less contribution to E_0^+ amplitude, whose estimation strongly depends on nucleon size, and that the longitudinal contribution from Σ_{space}^a term does not affect L_0^+ amplitude due to the artificial gauge invariance constraint.

B. The time component of Σ term

If one takes the conservation of vector current (CVC), equal time commutator $[Q_a^5, V_\mu(y)]_{x_0=y_0} = i\epsilon_{a3b} A_\mu^b(y)$, where $Q_a^5 = \int d^3x A_0^a(x)$, and its model independent deriva-

tive form $[D^a(x), V_0(0)] = i\epsilon_{a3b}D^b(0)$, the time component of eq.(8) reduces to the following nucleon expectation value :

$$\begin{aligned} \frac{iq^0 \Sigma_0^a(\gamma^* \pi^a)}{m_\pi^2 f_\pi} &= \frac{iq^0}{m_\pi^2 f_\pi} i\epsilon_{a3b} \langle p_2 | \partial^\mu A_\mu^b(0) | p_1 \rangle \\ &= -\frac{q^0}{f_\pi} \bar{u}(p_2) \gamma_5 \frac{I_a}{2} \frac{G_P(t)}{2M} u(p_1) , \end{aligned} \quad (11)$$

where we used eqs.(1) and (2) for PCAC and pion source function. The isospin structure $I_a = i\epsilon_{a3b}\tau_b$ means no contribution of this term to π^0 production likewise to C_a^ν term in eq.(6). Here the dependence on the nucleon model is replaced with the form factor of $G_P(t)$. It means that one does not need to consider nucleon model if one uses the experimental form factors of $G_A(t)$ [2] and assumes the pion pole dominance for $G_P(t)$ [10,13],

$$G_P(t) = \frac{4M^2}{m_\pi^2 - t} G_A(t) , \quad G_A(t) = g_A/(1 - t/\Lambda^2)^2 , \quad \Lambda \sim 1\text{GeV} . \quad (12)$$

The momentum transfer at the pion threshold is given as $t = (k^2 - m_\pi^2)/(1 + \mu)$ and $\mu = m_\pi/M$. Addition of this term to induced PS part, the 2nd term in $C_{\nu=0}^a/f_\pi$ of eq.(6), gives

$$\begin{aligned} &\frac{1}{f_\pi} \bar{u}(p_2) \frac{I_a}{2} \frac{G_P(t)}{2M} (k_0 - 2q_0) \gamma_5 u(p_1) \\ &= \frac{1}{f_\pi} \bar{u}(p_2) I_a \frac{M G_A(t)}{(m_\pi^2 - t)} (k_0 - 2q_0) \gamma_5 u(p_1) \\ &= g_{\pi NN} \bar{u}(p_2) I_a \frac{1}{t - m_\pi^2} (2q_0 - k_0) \gamma_5 u(p_1) , \end{aligned} \quad (13)$$

This equals to the time component of t-pole in Born terms i.e. the t-pole contribution at the pion threshold can be fully accounted for the sum of σ -pole and h-pole in CA approach. Moreover if we calculate transition amplitude by multiplying photon polarization, the t-pole contribution at the pion threshold is exactly double value of the σ -pole contribution.

$$\begin{aligned} &g_{\pi NN} \bar{u}(p_2) I_a \frac{1}{t - m_\pi^2} \epsilon^\mu (2q_\mu - k_\mu) \gamma_5 u(p_1) \\ &= g_{\pi NN} \bar{u}(p_2) I_a \frac{1}{t - m_\pi^2} \epsilon^0 (2q_0) \gamma_5 u(p_1) = 2 \epsilon^0 \frac{iq^0}{\mu^2 f_\pi} \Sigma_0^a(\gamma^* \pi^a) . \end{aligned} \quad (14)$$

It demonstrates that half of the t-pole contribution at the pion threshold is just the chiral symmetry breaking effects due to the time component of Σ term in pion eletroproduction,

and another half comes from the induced PS part in C_ν^a term. Of course this can be rewritten as a pseudo-vector (PV) coupling form if we use the Dirac equation and the equivalence relation between PS and PV coupling constants for πNN interaction. Schaefer and Weise [11] claimed that this Σ_0^a contribution is equal to the t-pole in Born terms, but it should be just half of t-pole as shown above.

However it is uncertain [2] whether the above relations can be hold generally beyond threshold, because we need to take the model dependent, spatial component $\Sigma_i^a(\gamma N)$ [11] from commutator $[\partial^\mu A_\mu^a(x), V_i^\gamma(0)]$ into account as discussed already.

C. The estimation of Σ_0^a

Before discussing the physical meaning of this chiral symmetry breaking term, we can extract a value for this term. From eq.(11), one can simply get the following result :

$$i\Sigma_0^\alpha(\gamma^*\pi^{-\alpha}) = -\frac{m_\pi^2}{2M} \frac{G_P(t)}{\sqrt{2M(E_1 + M)}} < \frac{[\tau_\alpha, \tau_3]}{4} > \chi_2^+ \vec{\sigma} \cdot \vec{p}_1 \chi_1 . \quad (15)$$

Taking an average value for the spins of initial and final nucleons

$$i\bar{\Sigma}_0^\alpha(\gamma^*\pi^{-\alpha}) = \frac{m_\pi^2}{2\pi M} \sqrt{\frac{(W - M)^2 - k^2}{4WM}} G_P(t) < \frac{[\tau_\alpha, \tau_3]}{2} > , \quad (16)$$

and eq.(12) for the induced PS form factor and the experimental $G_A(t)$ form factor, we obtain 38.1 MeV as the value of $|i\bar{\Sigma}_0^\alpha(\gamma^*\pi^\pm)|$ at the γ point i.e. $k^2 = 0$ point. The physical implication of this value is discussed at the next section with explanations for another chiral symmetry breaking terms, the σ term in $\pi - N$ scattering, and the Σ_{space}^a term in π^0 photoproduction.

III. CHIRAL SYMMETRY BREAKING

The chiral symmetry breaking effect in the $\pi - N$ scattering, so called σ term, gives a well known result [21] about the scalar quark density distribution of u and d quarks in nucleon,

$$\begin{aligned}
\sigma_{\pi N}^{ab} &= i \int d^4x e^{iqx} \langle p_2 | \delta(x_0) [A_0^a, \partial^\mu A_\mu^b(x)] | p_1 \rangle \\
&= \langle p_2 | [Q^{5a}, [Q^{5b}, H]] | p_1 \rangle \\
&= \frac{1}{2} (m_u + m_d) \delta_{ab} \langle p_2 | (\bar{u}u + \bar{d}d) | p_1 \rangle .
\end{aligned} \tag{17}$$

Here the following facts are used : the spatial divergence of axial current vanishes on integration over all space, and the axial charges Q^{5a} are conserved in the absense of chiral symmetry breaking hamiltonian H_{int} . The total hamiltonian $H = H_0 + H_{int}$, where H_0 is $SU(3)_L \otimes SU(3)_R$ invariant hamiltonian and H_{int} is the explicit chiral symmetry breaking term due to quark masses given by

$$H_{int} = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s . \tag{18}$$

If we use the Zweig sum rule from $SU(3)$ symmetry, this $\sigma_{\pi N}$ is related to the ratio of $\langle N | \bar{s}s | N \rangle$ and $\langle N | \bar{u}u + \bar{d}d | N \rangle$ [21]. Therefore the experimental value [30] of $\sigma_{\pi N}$, which lies on $45 \sim 60$ MeV, gives a valuable insight of quark scalar density $\bar{s}s$ as well as $\bar{u}u + \bar{d}d$ contribution in nucleon.

Using the same method as the σ term in $\pi - N$ scattering we show that $\Sigma_{space}^a(\gamma N)$ can be expressed as the expectation value of a quark tensor current,

$$\begin{aligned}
i\Sigma_j^a(\gamma N) &= i \int d^4x e^{iqx} \langle p_2 | \delta(x_0) [\partial^\mu A_\mu^a(x), V_j^\gamma] | p_1 \rangle \\
&= \int d^3x \langle p_2 | [[Q^{5a}, H_{int}], V_j^\gamma(\vec{x})] | p_1 \rangle \\
&= -\frac{1}{2} (m_u + m_d) \langle p_2 | \delta_{a3} (\bar{u}F_j u + \bar{d}F_j d) | p_1 \rangle, \text{ with } F_j = \frac{1}{2} \epsilon_{jkl} \sigma_{kl} .
\end{aligned} \tag{19}$$

This symmetry breaking term, survived in pion photoproduction, should be gauge invariant by itself because of the manifest gauge invariance of Born terms for photoproduction as reported by Schaefer and Weise [11]. The redundant term from this condition leads to the cancellation of the longitudinal part in Σ_{space}^a term.

However the time component of sigma term, Σ_0^a , can be given fully model independently as the nucleon expectation value of the axial current divergence (see eq.(11)). But here instead of using phenomenological form factors we represent it in terms of quarks using the same techniques as σ term in $\pi - N$ scattering :

$$\begin{aligned}
i\Sigma_0^a(\gamma^*\pi^a) &= i \int d^4x e^{iqx} \langle p_2 | \delta(x_0) [\partial^\mu A_\mu^a(x), V_0^\gamma] | p_1 \rangle \\
&= \int d^3x \langle p_2 | [[Q^{5a}, H_{int}], V_0^\gamma(\vec{x})] | p_1 \rangle \\
&= \frac{1}{2}(m_u + m_d) \langle p_2 | \epsilon_{a3c} v_c | p_1 \rangle, \quad \text{with } v_c = -i\bar{q}\lambda_c\gamma^5 q.
\end{aligned} \tag{20}$$

This can be rewritten for real pion representation :

$$\begin{aligned}
i\Sigma_0^\alpha(\gamma^*\pi^{-\alpha}) &= -\frac{1}{2}(m_u + m_d) \langle p_2 | \bar{q}_i \gamma_5 \frac{[\tau_\alpha, \tau_3]}{2} q_i | p_1 \rangle \\
&= \frac{(m_u + m_d)}{\sqrt{2}} \langle p_2 | -\bar{d} \gamma_5 u | p_1 \rangle \quad \text{for } (\gamma^*\pi^+), \\
&\text{or } \frac{(m_u + m_d)}{\sqrt{2}} \langle p_2 | \bar{u} \gamma_5 d | p_1 \rangle \quad \text{for } (\gamma^*\pi^-).
\end{aligned} \tag{21}$$

Therefore provided that theoretical or experimental results are known, one can extract some information for the charged PS quark density distribution in nucleon. For example our simple estimation for the above terms in section II leads to 38.1 MeV. The possibility exploiting experimental results is discussed in next section.

Neutral PS quark density $\langle p_2 | \bar{u} \gamma_5 u + \bar{d} \gamma_5 d | p_1 \rangle$ can be also obtained if the singlet currents V_μ^0 and A_μ^0 in $U(1)_V$ and $U(1)_A$ gauge are taken into account, for instance, in weak pionproduction. But here we deal with charged PS density since we consider only the electro-magnetic production of pion on nucleon.

IV. EXTRACTION OF Σ TERM FROM L_0^+ AMPLITUDE

As discussed already, the contribution of $\Sigma_0^a(\gamma^*N)$ to transition amplitude for electroproduction is a half of t-pole contribution, thus partially contributes to L_0^+ amplitude. Here we estimate what to extent it affects L_0^+ amplitude. We consider the general hadronic transition current matrix element [10,22] for the electroproduction of pion on nucleon. They are given by

$$\epsilon_\mu \mathcal{M}^\mu = \epsilon_\mu \sum_{i=1}^8 \bar{u}(p_2) Q_i^\mu A_i(\nu, \nu_1, \nu_2) u(p_1), \tag{22}$$

where $\nu = (P_i + P_f) \cdot k/2M^2$, $\nu_1 = q \cdot k/2M^2$, $\nu_2 = k^2/M^2$ and the $u(p_1, p_2)$ are the Dirac spinors of initial and final nucleons. The eight covariant operators Q_i^μ are given as follows :

$$\begin{aligned}
Q_1^\mu &= \gamma_5 \gamma^\mu, & Q_2^\mu &= \gamma_5 \frac{(P_i + P_f)^\mu}{4M}, \\
Q_3^\mu &= \gamma_5 \frac{q^\mu}{2M}, & Q_4^\mu &= \gamma_5 \frac{k^\mu}{2M}, \\
Q_5^\mu &= \gamma_5 \frac{\gamma^\mu \gamma \cdot k - \gamma \cdot k \gamma^\mu}{4M}, & Q_6^\mu &= \gamma_5 \gamma \cdot k \frac{(P_i + P_f)^\mu}{8M^2}, \\
Q_7^\mu &= \gamma_5 \gamma \cdot k \frac{q^\mu}{4M^2}, & Q_8^\mu &= \gamma_5 \gamma \cdot k \frac{k^\mu}{4M^2},
\end{aligned} \tag{23}$$

where k and q are the four momenta of initial photon and outgoing pion respectively. P_i and P_f are the momenta of initial and final nucleons. The invariant Ball amplitudes A_i are determined from all pole contributions to transition amplitudes in Born approximation or the chiral effective lagrangians in chiral perturbation theory [35]. Originally these amplitudes are constrained by the gauge invariance $k^\mu M_\mu = 0$, so that one has six independent invariant amplitudes. Those remained amplitudes can be expressed as CGLN amplitudes [24]. From them one can directly extract the following relation between the transition amplitudes and the E_0^+ and L_0^+ amplitudes near threshold using the specialized polarization photon vector [10], which has no time component,

$$\frac{-e}{4\pi(1+\mu)} \epsilon_\mu \mathcal{M}^\mu|_{thr.} = \chi_f^+ [E_0^+ \vec{\sigma} \cdot \vec{b} + L_0^+ \vec{\sigma} \cdot \hat{k} \hat{k} \cdot \vec{a}] \chi_i, \tag{24}$$

where $\vec{a} = \vec{\epsilon} - \frac{\epsilon_0}{k_0} \vec{k}$ and $\vec{b} = \vec{a} - \hat{k}(\hat{k} \cdot \vec{a}) = \vec{\epsilon} - \hat{k}(\hat{k} \cdot \vec{\epsilon})$. The result of L_0^+ amplitude in terms of A_i amplitude is well known

$$\begin{aligned}
E_0^+|_{thr.} &= -\frac{e}{2M} \left[\frac{M}{4\pi W} \sqrt{\frac{E_i + M}{2M}} \mathcal{A} \right]_{thr.}, \\
L_0^+|_{thr.} &= E_0^+|_{thr.} - \frac{e}{2M} \left[\frac{E_i - M}{2M} \frac{M}{4\pi W} \sqrt{\frac{E_i + M}{2M}} \mathcal{B} \right]_{thr.},
\end{aligned} \tag{25}$$

where $\mathcal{A} = A_1 + \frac{\mu}{2} A_5$, $\mathcal{B} = -\frac{1}{2} A_2 + A_4 - A_5 + \frac{1}{4} (2 + \mu) A_6 - \frac{1}{2} (2 + \mu) A_8$. But only A_3 and A_4 are survived for t-pole, which are calculated as follows :

$$A_3^{(-)} = -\frac{8gM^2 F_\pi(k^2)}{m_\pi^2 - t}, \quad A_4^{(-)} = \frac{4gM^2 F_\pi(k^2)}{m_\pi^2 - t}, \tag{26}$$

where the coupling constant $g_{\pi NN}^{PS}$ is designated as g . One may argue that there is additional term in $A_4^{(-)}$ from the gauge invariance of total amplitude. But its contribution is not taken

into account here because we do not need to maintain self gauge invariance in t-pole. The final value obtained this way is

$$\Delta L_0^+(\sigma - pole) = \left[\frac{eg}{2M} \frac{1}{4\pi(1+\mu)} \sqrt{\frac{(2+\mu)^2 - \nu_2}{4(1+\mu)}} \right] \frac{F_\pi(k^2)}{\sqrt{2}} \frac{\mu^2 - \nu_2}{\mu^2(2+\mu) - \nu_2} \text{ for } \gamma^* \pi^+. \quad (27)$$

If we compare to the following total L_0^+ amplitude [22,23], which was obtained from Born approximation,

$$L_0^+ = \left[\frac{eg}{2M} \frac{1}{4\pi(1+\mu)} \sqrt{\frac{(2+\mu)^2 - \nu_2}{4(1+\mu)}} \right] \cdot \sqrt{2} \left[F_\pi(k^2) \frac{\mu(\mu + \nu_2)}{\mu^2(2+\mu) - \nu_2} - G_E^n \frac{\nu_2}{2 - \nu_2} - \frac{1}{2}\mu \right] \text{ for } \gamma^* \pi^+, \quad (28)$$

and let

$$\alpha = \frac{\Delta L_0^+(\sigma - pole)}{L_0^+}, \quad (29)$$

then α_{theory} for $\gamma^* \pi^+$ at γ point is about $\frac{1}{2}$. Of course one can use more refined theory for L_0^+ amplitude to extract α_{theory} [6]. In order to directly measure this σ -pole contribution, $\Delta L_0^+(\sigma - pole)$, from the experiments one has to separate t-pole contribution from the whole amplitude. Such a separation is a formidable task in experimental side. Even the total L_0^+ amplitude for π^+ electroproduction is not measured yet despite of a try at Saclay [13]. Therefore the future experimental results for L_0^+ amplitude in π^+ electroproduction near threshold would give semi-empirical value for $\Delta L_0^+(\sigma - pole)$,

$$\Delta L_0^+(\sigma - pole)_{semi-emp.} = L_0^+(exp.) \cdot \alpha_{theory}. \quad (30)$$

If one uses this value at the r.h.s. of eq.(24), one can deduce Σ_0^a from the l.h.s.. Since the PS quark distribution on nucleon is closely related to this quantity, the $L_0^+(\sigma - pole)$ amplitude from the future would-be experiments could give invaluable informations about nucleon structure. Also it could be an important test for our theoretical results i.e. 38.1 MeV for $|i\bar{\Sigma}_0^\alpha(\gamma^* \pi^+)|$ (see eqs. (16) and (21)) in section II and III, from experiments.

V. CONCLUSION

The t-channel pole in Born terms contributes significantly to electroproduction. At the pion threshold such a t-pole contribution to transition amplitude is explained as a sum for those of h-pole and σ -poles in CA approach. Both contributions are shown to be a just half of t-pole. The former (h-pole) comes from the induced PS part in nucleon expectation value of axial current. The latter (σ -pole) is chiral symmetry breaking effect similar to σ term in $\pi-N$ scattering. It means that chiral symmetry breaking effect explicitly appears as a t-pole contribution to electroproduction at the pion threshold, thus gives a sizable contribution to L_0^+ amplitude.

From the viewpoint of quark density distribution on nucleon this chiral symmetry breaking represents a charged PS quark density, which is estimated as 38 MeV by exploiting the experimental form factors. The future experimental result for L_0^+ amplitude is expected to make some conclusive remarks for this quantity. Weak pionproduction can be a good guide about the neutral PS quark density distribution on nucleon.

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